UNIT TEST - 4
MATHEMATICS

CLASS XII **Time : 1½ hrs.**

SET - A **Mark : 40**

SECTION - A (OBJECTIVE TYPE QUESTIONS) **9 × 1 = 9**

1. General solution of $\frac{dy}{dx} + 2y = \sin x$ is

   a) $y = \frac{1}{5}(2\sin x + \cos x) - Ce^{-2x}$ b) $y = \frac{1}{5}(2\sin x + \cos x) + Ce^{-2x}$

   c) $y = \frac{1}{5}(2\sin x - \cos x) - Ce^{-2x}$ b) $y = \frac{1}{5}(2\sin x - \cos x) + Ce^{-2x}$

2. The degree of the different equation $\left(\frac{d^2y}{dx^2}\right)^2 - \left(\frac{dy}{dx}\right) = y^3$, is

   a) 4 b) $\frac{1}{2}$ c) 2 d) 3

3. The general solution of the different equation $(xdy - ydx)\tan\frac{y}{x} = nx^2\,dx$ is :

   a) $\sec\left(\frac{y}{x}\right) = ce^{nx}$ b) $\cot\left(\frac{y}{x}\right) = e^{nx+c}$

   c) $\sin\left(\frac{y}{x}\right) = ce^{nx}$ d) $\cos\left(\frac{y}{x}\right) = e^{nx+c}$

4. If xdy = y(dx + ydy), y(1) = 1 and y(x) > 0. Then, y(-3) is equal to

   a) 2 b) 1 c) 0 d) 3

5. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$

   a) $\frac{1}{3}\left(-160\hat{i} - 5\hat{j} + 70\hat{k}\right)$ b) $\frac{1}{3}\left(160\hat{i} - 5\hat{j} - 70\hat{k}\right)$

   c) $\frac{1}{3}\left(160\hat{i} + 5\hat{j} + 70\hat{k}\right)$ d) $\frac{1}{3}\left(160\hat{i} + 5\hat{j} - 70\hat{k}\right)$

6. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$

   a) $\vec{a} = -\frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$ b) $\vec{a} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{2}{\sqrt{6}}\hat{k}$

   c) $\vec{a} = \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} - \frac{2}{\sqrt{6}}\hat{k}$ d) $\vec{a} = \frac{1}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} - \frac{2}{\sqrt{6}}\hat{k}$

7. If $\vec{a}$ and $\vec{b}$, $\vec{a} \times \vec{b}$ for non-zero vectors is a unit vector and $|\vec{a}| = |\vec{b}| = \sqrt{2}$ two angle θ between vectors $\vec{a}$ and $\vec{b}$ is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{6}$ d) $\frac{-\pi}{2}$

8. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + \hat{j} - \hat{k}$. A vector coplanar to $\vec{a}$ and $\vec{b}$ has a projection along $\vec{c}$ of magnitude $\frac{1}{\sqrt{3},}$ then the vector is

a) $4\hat{i} - \hat{j} + 4\hat{k}$ b) None of these c) $4\hat{i} + \hat{j} - 4\hat{k}$ d) $2\hat{i} + \hat{j} + 4\hat{k}$

9. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that $|\vec{a} + \vec{b}| = \sqrt{3}$. If $\vec{c} = \vec{a} + 2\vec{b} + 3(\vec{a} \times \vec{b})$, then $2|\vec{c}|$ is

a) $\sqrt{55}$ b) $\sqrt{37}$ c) $\sqrt{51}$ d) $\sqrt{43}$

SECTION - B - (VERY SHORT ANSWER QUESTIONS)

10. Solve : $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ **2**

11. Find the general solution of $(x+y)\frac{dy}{dx} = 1$ **2**

OR

Show that $y = ax^3 + bx^2 + c$ is a solution of the different equation $\frac{d^3y}{dx^3} = 6a$

12. Write the position vector of a point dividing the line segment joining points having position vectors $\hat{i} + \hat{j} - 2k$ and $2\hat{i} - \hat{j} + 3k$ externally in the ratio 2 : 3. **2**

13. Find the area of the parallelogram whose diagonals are $3\hat{i} + 4\hat{j}$ and $\hat{i} + \hat{j} + \hat{k}$ **2**

14. If $\vec{a}$ and $\vec{b}$ are two unit vectors and θ is angle between them, then show that $\frac{1}{2}(\vec{a} - \vec{b})^2 = 1 - \cos\theta.$ **2**

SECTION - C - (SHORT ANSWER QUESTIONS)

15. Show that the points $(2\hat{i} - \hat{j} + \hat{k})$, $(\hat{i} - 3\hat{j} - 5\hat{k})$, $(3\hat{i} - 4\hat{j} - 4\hat{k})$ from the vertices of a right-angled triangle. **3**

16. Show that the four points P, Q, R, S with position vectors $\vec{p}, \vec{q}, \vec{r}, \vec{s}$ respectively such that $5\vec{p} - 2\vec{q} + 6\vec{r} - 9\vec{s} = \vec{0}$, are coplanar. Also, find the position vector of the point of intersection of the line segments PR and QS. **3**

17. Find the particular solution of the differential equation $2ye^{x/y}\,dx + (y - 2xe^{x/y})\,dy = 0$, given that x = 0, when y = 1. **3**

OR

Find the particular solution of the differential equation $\left[x\sin^2\left(\frac{y}{x}\right) - y\right]dx + x\,dy = 0$, given that $y = \frac{\pi}{4}$ when x = 1.

18. $\vec{a}, \vec{b}, \vec{c}$ are unit vectors, suppose $\vec{a}\cdot\vec{b} = \vec{a}\cdot\vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$. Prove that $\vec{a} = \pm 2\left(\vec{b}\times\vec{c}\right)$. **3**

SECTION - D - (LONG ANSWER QUESTIONS)

19. Solve the differential equation $\cos^2 x\frac{dy}{dx} + y = \tan x$ **5**

OR

Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{p}$, which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{p}.\vec{c} = 18$

SECTION - E - (CASE BASED QUESTIONS)

20. A veterinary doctor was examining a sick cat brought by a pet lover. When it was brought to the hospital, it was already dead. The pet lover wanted to find its time of death. He took the temperature of the cat at 11.30 pm which was 94.6^0 F. He took the temperature again after one hour; the temperature was lower than the first observation. It was 93.4^0 F. The room in which the cat was put is always at 70^0 F. The normal temperature of the cat was 98.6^0 F when it was alive. The doctor estimated the time of death using Newton law of cooling which is governed by the differential equation : $dT/dt \;\alpha\; (T - 70)$, where 70^0 F is the room temperature and T is the temperature of the object at time t. Substituting the two different observations of T and t made, in the solution of the differential equation : $dT/dt = K(T - 70)$ where k is a constant of proportion, time of death is calculated.

i) State the degree of the above given differential equation. **1**

ii) Which method of solving a differential equation helped in calculation of the time of death? **1**

iii) Find the solution of the differential equation $dT/dt = K(T - 70)$. **2**

UNIT TEST - 4

MATHEMATICS

CLASS XII **Time : 1½ hrs.**

SET - B **Mark : 40**

SECTION - A (OBJECTIVE TYPE QUESTIONS) **$9 \times 1 = 9$**

1. What is the solution of the differential equation $\frac{dx}{dy} + \frac{x}{y} - y^2 = 0$?

   a) $xy = x^4 + C$ b) $3xy = y^3 + C$ c) $xy = y^4 + C$ d) $4xy = y^4 + C$

2. What is integrating factor $\frac{dy}{dx}$ + y sec x = tan x

   a) sec x + tan x b) log(sec x + tan x) c) $e^{\sec x}$ d) sec x

3. The differential equation $\frac{dy}{dx}$ + Py = Qy^{11}, n > 2 can be reduced to linear form by substituting

   a) $z = y^{1-n}$ b) $z = y^{n-1}$ c) $z = y^{n+1}$ d) $z = y^n$

4. The solution of the differential equation $y\,dx - (x + 2y^2)\,dy = 0$ is $x = f(y)$. If $f(-1) = 1$, then $f(1)$ is equal to :

   a) 2 b) 4 c) 3 d) 1

5. If $\vec{a}.\vec{b} = \vec{a}.\vec{c}$ and $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}, \vec{a} = 0,$ then

   a) $\vec{b} + \vec{c} = \vec{0}$ b) none of these c) $\vec{b} = \vec{c}$ d) $\vec{b} = \vec{0}$

6. ABCD is a parallelogram with AC and BD as diagonals. Then, $\overrightarrow{AC} - \overrightarrow{BD} =$

   a) $3\overrightarrow{AB}$ b) $\overrightarrow{AB}$ c) $4\overrightarrow{AB}$ d) $2\overrightarrow{AB}$

7. The value of λ for which the vectors $3\hat{i} - 6\hat{j} + \hat{k}$ and $2\hat{i} - 4\hat{j} + \lambda\hat{k}$ are parallel is

   a) $\frac{5}{2}$ b) $\frac{2}{5}$ c) $\frac{2}{3}$ d) $\frac{3}{2}$

8. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}, \vec{b} = \hat{i} - \hat{j} + \hat{k}, \vec{c} = \hat{i} + \hat{j} - \hat{k}.$ A vector coplanar to $\vec{a}$ and $\vec{b}$ has a projection along $\vec{c}$ of magnitude $\frac{1}{\sqrt{3},}$ then the vector is

   a) $4\hat{i} - \hat{j} + 4\hat{k}$ b) None of these c) $4\hat{i} + \hat{j} - 4\hat{k}$ d) $2\hat{i} + \hat{j} + 4\hat{k}$

9. Find a unit vector in the direction $\vec{a} = 3\hat{i} - 2j + 6\hat{k}$

a) $\frac{3}{7}\hat{i} - \frac{2}{7}\hat{j} + \frac{6}{7}\hat{k}$

b) $6\hat{i} + 2\hat{k}$

c) $\frac{6}{5}\hat{i} + \frac{2\hat{j}}{k} + 5\hat{k}$

d) $\frac{7}{4}\hat{i} + \frac{3}{2}\hat{j} + \frac{4}{3}\hat{k}$

SECTION - B - (VERY SHORT ANSWER QUESTIONS)

10. Solve differential equation : $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$ **2**

11. Find the general solution for differential equation : $\frac{dy}{dx} = e^{x+y} + e^{x-y}$ **2**

OR

Verify that y = $ce^{\tan^{-1}x}$ is a solution of the different equation $\left(1+x^2\right)\frac{d^2y}{dx^2} + \left(2x - 1\right)\frac{dy}{dx} = 0$

12. Find $\left|\vec{a} \times \vec{b}\right|$, if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ **2**

13. Show that the points $2\hat{i},\ -\hat{i} - 4\hat{j}$ and $-\hat{i} + 4\hat{j}$ form an isosceles triangles. **2**

14. Show that the solution of differential equation y = $2(x^2 - 1) + ce^{-x^2}$ is $\frac{dy}{dx} + 2xy = 4x^3$. **2**

SECTION - C - (SHORT ANSWER QUESTIONS)

15. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k},\ \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c}.\vec{d} = 15$. **3**

16. Prove that the points having positions vectors $\hat{i} + 2\hat{j} + 3\hat{k}, 3\hat{i} + 4\hat{j} + 7\hat{k}, -3\hat{i} - 2\hat{j} - 5\hat{k}$ are collinear. **3**

17. Find the particular solution of the differential equation $(1 - y^2)(1 + \log|x|)\,dx + 2xy\ dy = 0$ given that y = 0, when x = 1. **3**

OR

Find the general solution of the differential equation d $x\frac{dy}{dx} + y - x + xy\cot x = 0,\ x \neq 0.$

18. If $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k},\ \vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$, show that $\left(\vec{a} + \vec{b}\right)$ and $\left(\vec{a} - \vec{b}\right)$ are perpendicular to each other. **3**

SECTION - D - (LONG ANSWER QUESTIONS)

19. Show that the differential equation $\left(1+e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy = 0$ is homogeneous and solve it. **5**

OR

Find the position vector of a point R, which divides the line joining two points P and Q whose position vectors are $2\vec{a} + \vec{b}$ and $\vec{a} - 3\vec{b}$ respectively, externally in the ratio 1 : 2. Also, show that P is the mid-point of line segment RQ.

SECTION - E - (CASE BASED QUESTIONS)

20. Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation dy/dx = K (50 – y) where x denoted the number of weeks and y the number of children who have been given the drops.

i) State the order of the above given differential equation. **1**

ii) Which method of solving a differential equation can be used to solve dy/dx = K (50 – y)? **1**

iii) Find the solution of the differential equation dy/dx = K (50 – y). **2**