# MODEL EXAMINATION 2022-23

## Mathematics (Code-041)

**Time Allowed: 3 Hours** **Maximum Marks: 80**

***General Instructions :***

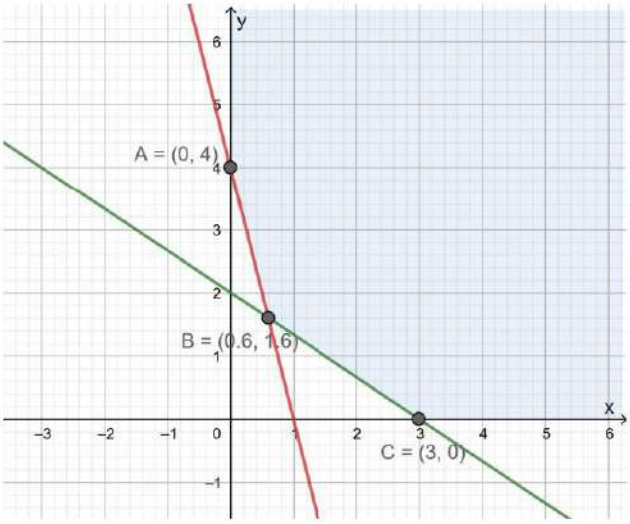
1. This Question paper contains - **five sections** A, B, C, D and E. Each section iscompulsory. However, there are internal choices in some questions.
2. **Section A** has 18 **MCQ's and 02** Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 **Very Short Answer (VSA)-type** questions of 2 marks each.
4. **Section C** has 6 **Short Answer (SA)-type** questions of 3 marks each.
5. **Section D** has 4 **Long Answer (LA)-type** questions of 5 marks each.
6. **Section E** has 3 **source based/case based/passage based/integrated units ofassessment (**4 marks each**)** with sub parts.

---

SECTION A

**(Multiple Choice Questions) Each question carries 1 mark**

| | |
|---|---|
| 1 | If A and B are symmetric matrices of same order, then AB-BA is a<br>(a) Skew-symmetric matrix (b) Symmetric matrix<br>(c) Zero matrix (d) Identity |
| 2 | If A $=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$, then the value of k if, $A^2 = kA - 2I$ is<br>(a) 0 (b) 8 (c) – 7 (d) 1 |
| 3 | The area of a triangle with vertices A, B, C is given by<br>(a) $\frac{1}{2}\|\overrightarrow{A\ B} \times \overrightarrow{A\ C}\|$ (b) $\frac{3}{4}\|\overrightarrow{A\ B} \times \overrightarrow{A\ C}\|$<br>(b) $\|\overrightarrow{A\ C} \times \overrightarrow{A\ B}\|$ – (d) $\frac{1}{2}\|\overrightarrow{A\ C} \times \overrightarrow{A\ B}\|$ |
| 4 | . The function $f(x) = e^{\|x\|}$ is<br>(a)Continuous everywhere but not differentiable at x=0<br>(b)Continuous and differentiable everywhere<br>(c)Not continuous at x=0<br>(d)None of the above |
| 5 | Find the antiderivative F of function defined by f(x) = $4x^3 - 6$, where F(0)= 3.<br>(a) $4x^4 - 6x + 3$ (b) $x^4 - 6x$ (c) $x^4 - 6x + 3$ (d) $x^4 - 6x - 3$ |
| 6 | Find order and degree of the differential equation: $x^2 \frac{d^2y}{dx^2} = \left\{1 + \left(\frac{dy}{dx}\right)^2\right\}^4$<br>(a) Order 2 Degree 1 (b) Order 1 Degree 2<br>(c) Order 4 Degree 1 (d) Order 1 Degree 4 |

| | |
|---|---|
| 7 | The solution set of the inequality $3x + 5y < 4$ is<br>(a) an open half-plane not containing the origin.<br>(b) an open half-plane containing the origin.<br>(c) the whole *XY*-plane not containing the line $3x + 5y = 4$.<br>(d) a closed half plane containing the origin. |
| 8 | The projection of the vector $3\hat{\imath} - \hat{\jmath} - 2\hat{k}$ *on the vector* $\hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ is<br>(a) $\frac{7}{14}$ (b) $\frac{7}{\sqrt{14}}$ (c) $\frac{6}{13}$ (d) $\frac{7}{2}$ |
| 9 | $\int_0^1 x\,(1-x)^4 dx =$<br>$(a)\frac{1}{20}$ (b) $\frac{1}{25}$ (c) $\frac{1}{30}$ (d) $\frac{1}{40}$ |
| 10 | If A and B are invertible matrices, then which of the following is not correct?<br>(a) adj $A = \lvert A\rvert . A^{-1}$ (b) $\det(A^{-1}) = (\det(A))^{-1}$<br>(c) $(AB)^{-1} = B^{-1}A^{-1}$ (d) $(A+B)^{-1} = B^{-1} + A^{-1}$ |
| 11 | The corner points of the shaded unbounded feasible region of an LPP are (0, 4), (0.6, 1.6) and (3, 0) as shown in the figure. The minimum value of the objective function $Z = 4x + 6y$ occurs at<br>  <br><br>(a)(0.6, 1.6) *only* (b) (3, 0) only (c) (0.6, 1.6) and (3, 0) only<br>(d) at every point of the line-segment joining the points (0.6, 1.6) and (3, 0) |
| 12 | The area of a triangle with vertices $(-3,0), (3,0)$ and $(0,k)$ is 9 sq. units. Then, the value of k will be<br>(a) 9 (b) 3 (c) -9 (d) 6 |
| 13 | Adjoint of matrix $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ is<br>(a) $\begin{bmatrix}4 & 2\\ 3 & 1\end{bmatrix}$ (b) $\begin{bmatrix}4 & -2\\ -3 & 1\end{bmatrix}$ (c) $\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ d) $\begin{bmatrix}1 & -2\\ -3 & 4\end{bmatrix}$ |
| 14 | Let E and F be events with P(E) = 3 /5 , P(F) =3/ 10 and P (E ∩ F) = 1/5 ,then<br>(a) E and F are independent (b) E and F are dependent<br>(c ) can't be determined (d) none of them |

| | |
|---|---|
| 15 | Find the equation of the curve passing through the point (1, 1) whose differential equation is $x\,dy = (2x^2 + 1)\,dx$ $(x \neq 0)$<br>(a) $y = x^2 + \log x$ (b) $y = x + \log x^2$ (c) $y = x^2 - \log x$ (d) $y = x - \log x$ |
| 16 | If $y = \log\sqrt{\tan x}$, then $\frac{dy}{dx}$ is<br>(A)$\cos 2x$ (B)$\sin 2x$ (C)$cosec\ 2x$ (D)none of these |
| 17 | If a is a nonzero vector of magnitude 'a' and λ a nonzero scalar, then λa is unit vector if<br>(A) $\lambda = 1$ (B) $\lambda = -1$ (C) $a = \lvert\lambda\rvert$ (D) $a = 1/\lvert\lambda\rvert$ |
| 18 | P is a point on the line joining the points $(0,5,-2)$ and $B(3,-1,2)$. If the x-coordinate of P is 6, then its z-coordinate is<br>(a) 10 (b) 6 (c) -6 (d) -10 |

**ASSERSTION REASON**

In the following questions, a statement of assertion (A) is followed by a statement ofReason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

| | |
|---|---|
| 19 | **Assertion (A)**Range of $\sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$<br>**Reason(R)**The principal value of $\sin^{-1}\frac{1}{2} = \frac{\pi}{6}$ |
| 20 | **Assertion (A)**: The acute angle between the line $\bar{r} = \hat{\imath} + \hat{\jmath} + 2\hat{k} + \lambda(\hat{\imath} - \hat{\jmath})$ and the x-axis is 45<br>**Reason(R)**: The acute angle $\theta$ between the lines<br>$\bar{r} = x_1\hat{\imath} + y_1\hat{\jmath} + z_1\hat{k} + \lambda(a_1\hat{\imath} + b_1\hat{\jmath} + c_1\hat{k})$ and<br>$\bar{r} = x\ \hat{\imath} + y\ \hat{\jmath} + z\ \hat{k} + \mu(\hat{a}\hat{\imath} + b\ \hat{\jmath} + c\ \hat{k})$ is given by<br>Cos θ = |

## SECTION B

***This section comprises of very short answer type-questions (VSA) of 2 marks each***

| | |
|---|---|
| 21 | Find the value of sin ( $\tan^{-1}$ (0.75))<br>Or<br>If N be the set of all-natural numbers, consider f: N → N such that f(x) = 2x, ∀ x ∈ N, then show that f is one-one but not onto. |

| | |
|---|---|
| 22 | A stone is dropped into a quiet lake and waves move in circles at a speed of 4cm per second. At the instant, when the radius of the circular wave is 10 cm, how fast is the enclosed area increasing? |
| 23 | Show that the points A (1, 2, 7), B (2, 6, 3) and C (3, 10, –1) are collinear.<br>0R<br>Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the YZ-plane. |
| 24 | If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2 y_2 + xy_1 + y = 0$ |
| 25 | Find the magnitude of two vectors , having the same magnitude and such that the angle between them is $60^0$ and their scalar product is $\frac{1}{2}$ |

## SECTION C

**(This section comprises of short answer type questions (SA) of 3 marks each)**

| | |
|---|---|
| 26 | Find $\int \frac{x^2+1}{x^2-5x+6} dx$ |
| 27 | A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale<br>OR<br>A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails. |
| 28 | Evaluate: $\int_0^{\frac{\pi}{2}} \sqrt{sin\emptyset} \, . cos^5 \emptyset \, d\emptyset$<br>OR<br>Evaluate: $\int_0^1 x\,(1-x)^n dx$ |
| 29 | Show that the differential equation (x – y) dy/ dx = x + 2y is homogeneous and solve it.<br>OR<br>Solve: $x^2 \frac{dy}{dx} = x^2 - 2y^2 + xy$ |
| 30 | Solve the following Linear Programming Problem graphically:<br>Maximize Z = 400x + 300y subject to $x + y \leq 200, x \leq 40, x \geq 20, y \geq 0$ |
| 31 | Evaluate: $\int \frac{x^2}{(x^2+1)(x^2+4)} dx$ |

SECTION D (LONG ANSWER TYPE) ( 5 MARKS EACH)

| | |
|---|---|
| 32 | Make a rough sketch of the region $\{(x, y): 0 \leq y \leq x^2, 0 \leq y \leq x, 0 \leq x \leq 2\}$ and find the area of the region using integration. |

| | |
|---|---|
| 33 | Show that the relation R defined in the set A of all polygons as R = {(P1, P2 ) : P1 and P2 have same number of sides}, is an equivalence relation. What is the set of all elements in A related to the right-angle triangle T with sides 3, 4 and 5?<br>OR<br>If R1 and R2 are equivalence relations in a set A, show that R1 ∩ R2 is also an equivalence relation. |
| 34 | An insect is crawling along the line $\bar{r} = 6\imath + 2\jmath + 2\hat{k} + (\imath - 2\jmath + 2\hat{k})$ and another insect is crawling along the line $\bar{r} = -4\imath - \hat{k} + (3\imath - 2\jmath - 2\hat{k})$. At what points on the lines should they reach so that the distance between them is the shortest? Find the shortestpossible distance between them. |
| 35 | Solve the system of equations by using matrix method:<br>$2x - 3y + 5z = 11$<br>$3x + 2y - 4z = -5$<br>$x + y - 2z = -3$ |

SECTION E( CASE BASED QUESTIONS)

**(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-parts. First two case study questions have three sub-parts (i), (ii), (iii)of marks 1, 1, 2 respectively. The third case study question has two sub-parts of 2 marks each.)**

| | |
|---|---|
| 36 | Shape of a toy is given as $f(x) = 6(2x^4 - x^2)$. To make the toy beautiful 2 sticks which are perpendicular to eachother were placed at a point (2,3), above the toy.<br>**1.** Which value from the following may be abscissa of critical point?<br>**2.** Find the second order derivative of the function at x = 5.<br>**3.** At which of the following intervals will f(x) be increasing? |
| 37 | $P(x) = -5x^2 + 125x + 37500$ is the total profit function of acompany, where x is the production of the company.<br>**1.** What will be the production when the profit is maximum?<br>**2.** What will be the maximum profit?<br>Check in which interval the profit is strictly increasing.<br><br>When the x value lies between (2,3) then the function is |

| | |
|---|---|
| 38 |  <br>There are two antiaircraft guns, named as A and B. The probabilities that the shell firedfrom them hits an airplane are 0.3 and 0.2 respectively. Both of them fired one shell at an airplane at the same time.<br>(i) What is the probability that the shell fired from exactly one of them hit the plane?<br>(ii) If it is known that the shell fired from exactly one of them hit the plane, then what isthe probability that it was fired from B? |